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Baccalauréat International
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MARKSCHEME

May 2014

MATHEMATICS

Higher Level

Paper 1

24 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to Scoris instructions and the document "**Mathematics HL: Guidance for e-marking May 2014**". It is essential that you read this document before you start marking. In particular, please note the following:

- Marks must be recorded using the annotation stamps. Please check that you are entering marks for the right question.
- If a part is **completely correct**, (and gains all the "must be seen" marks), use the ticks with numbers to stamp full marks.
- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, it **must** be recorded using **all** the annotations.
- All the marks will be added and recorded by Scoris.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc.*, do **not** split the marks.
- Once a correct answer to a question or part-question is seen, ignore further working.

3 *N* marks

*Award **N** marks for **correct** answers where there is **no** working.*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

4 Implied marks

Implied marks appear in ***brackets eg (M1)****, and can only be awarded if* ***correct*** *work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

5 Follow through marks

*Follow through (****FT****) marks are awarded where an incorrect answer from one* ***part*** *of a question is used correctly in* ***subsequent*** *part(s). To award* ***FT*** *marks,* ***there must be working present*** *and not just a final answer based on an incorrect answer to a previous part.*

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).
- Within a question part, once an error is made, no further **dependent** ***A*** marks can be awarded, but ***M*** marks may be awarded if appropriate.
- Exceptions to this rule will be explicitly noted on the markscheme.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (****MR****). A candidate should be penalized only once for a particular mis-read. Use the* ***MR*** *stamp to indicate that this has been a misread. Then deduct the first of the marks to be awarded, even if this is an* ***M*** *mark, but award all others so that the candidate only loses one mark.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).

7 Discretionary marks *(d)*

An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief ***note*** *written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

Unless the question specifies otherwise, ***accept*** *equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

Example: for differentiating $f(x) = 2\sin(5x-3)$, the markscheme gives:

$$f'(x) = \left(2\cos(5x-3)\right)5 \quad \left(=10\cos(5x-3)\right) \qquad \textbf{\textit{A1}}$$

Award ***A1*** for $\left(2\cos(5x-3)\right)5$, even if $10\cos(5x-3)$ is not seen.

10 Accuracy of Answers

Candidates should ***NO LONGER*** *be penalized for an accuracy error* ***(AP)****.*

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures. Please check work carefully for ***FT****.*

11 Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

12 Calculators

No calculator is allowed. The use of any calculator on paper 1 is malpractice, and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice. Examples: finding an angle, given a trig ratio of 0.4235.

13 More than one solution

Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise.

14. Candidate work

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. This work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

SECTION A

1. (a) $\mathrm{P}(A \cap B) = \mathrm{P}(A \mid B) \times P(B)$

$\mathrm{P}(A \cap B) = \frac{2}{11} \times \frac{11}{20}$ ***(M1)***

$= \frac{1}{10}$ ***A1***

[2 marks]

(b) $\mathrm{P}(A \cup B) = \mathrm{P}(A) + P(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cup B) = \frac{2}{5} + \frac{11}{20} - \frac{1}{10}$ ***(M1)***

$= \frac{17}{20}$ ***A1***

[2 marks]

(c) No – events *A* and *B* are not independent ***A1***

EITHER

$\mathrm{P}(A \mid B) \neq \mathrm{P}(A)$ ***R1***

$\left(\frac{2}{11} \neq \frac{2}{5}\right)$

OR

$\mathrm{P}(A) \times P(B) \neq \mathrm{P}(A \cap B)$

$\frac{2}{5} \times \frac{11}{20} = \frac{11}{50} \neq \frac{1}{10}$ ***R1***

Note: The numbers are required to gain ***R1*** in the '**OR**' method only.

Note: Do not award ***A1R0*** in either method.

[2 marks]

Total [6 marks]

2. **METHOD 1**

$2^{3(x-1)} = (2\times 3)^{3x}$ ***M1***

Note: Award ***M1*** for writing in terms of 2 and 3.

$2^{3x}\times 2^{-3} = 2^{3x}\times 3^{3x}$

$2^{-3} = 3^{3x}$ ***A1***

$\ln\left(2^{-3}\right) = \ln\left(3^{3x}\right)$ ***(M1)***

$-3\ln 2 = 3x\ln 3$ ***A1***

$x = -\dfrac{\ln 2}{\ln 3}$ ***A1***

METHOD 2

$\ln 8^{x-1} = \ln 6^{3x}$ ***(M1)***

$(x-1)\ln 2^3 = 3x\ln(2\times 3)$ ***M1A1***

$3x\ln 2 - 3\ln 2 = 3x\ln 2 + 3x\ln 3$ ***A1***

$x = -\dfrac{\ln 2}{\ln 3}$ ***A1***

METHOD 3

$\ln 8^{x-1} = \ln 6^{3x}$ ***(M1)***

$(x-1)\ln 8 = 3x\ln 6$ ***A1***

$x = \dfrac{\ln 8}{\ln 8 - 3\ln 6}$ ***A1***

$x = \dfrac{3\ln 2}{\ln\left(\dfrac{2^3}{6^3}\right)}$ ***M1***

$x = -\dfrac{\ln 2}{\ln 3}$ ***A1***

Total [5 marks]

3. (a) **EITHER**

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & -2 \\ 3 & -1 & 14 & 6 \\ 1 & 2 & 0 & -5 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 1 & 2 & -2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right)$$ ***M1***

row of zeroes implies infinite solutions, (or equivalent). ***R1***

Note: Award ***M1*** for any attempt at row reduction.

OR

$$\begin{vmatrix} 1 & 1 & 2 \\ 3 & -1 & 14 \\ 1 & 2 & 0 \end{vmatrix} = 0$$ ***M1***

$$\begin{vmatrix} 1 & 1 & 2 \\ 3 & -1 & 14 \\ 1 & 2 & 0 \end{vmatrix} = 0 \text{ with one valid point}$$ ***R1***

OR

$$x + y + 2z = -2$$
$$3x - y + 14z = 6$$
$$x + 2y = -5 \quad \Rightarrow x = -5 - 2y$$

substitute $x = -5 - 2y$ into the first two equations:

$-5 - 2y + y + 2z = -2$

$3(-5 - 2y) - y + 14z = 6$ ***M1***

$-y + 2z = 3$

$-7y + 14z = 21$

the latter two equations are equivalent (by multiplying by 7) therefore an infinite number of solutions. ***R1***

OR

for example, $7 \times R_1 - R_2$ gives $4x + 8y = -20$ ***M1***

this equation is a multiple of the third equation, therefore an infinite number of solutions. ***R1***

continued…

Question 3 continued

(b) let $y = t$ ***M1***

then $x = -5 - 2t$ ***A1***

$z = \frac{t+3}{2}$ ***A1***

OR

let $x = t$ ***M1***

then $y = \frac{-5-t}{2}$ ***A1***

$z = \frac{1-t}{4}$ ***A1***

OR

let $z = t$ ***M1***
then $x = 1 - 4t$ ***A1***
$y = -3 + 2t$ ***A1***

OR

attempt to find cross product of two normal vectors:

eg: $\begin{vmatrix} i & j & k \\ 1 & 1 & 2 \\ 1 & 2 & 0 \end{vmatrix} = -4i + 2j + k$ ***M1A1***

$x = 1 - 4t$
$y = -3 + 2t$
$z = t$ ***A1***
(or equivalent)

Total [5 marks]

4. (a) using the formulae for the sum and product of roots:

$\alpha+\beta=-2$ ***A1***

$\alpha\beta=-\frac{1}{2}$ ***A1***

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ ***M1***

$=(-2)^2-2\left(-\frac{1}{2}\right)$

$=5$ ***A1***

[4 marks]

Note: Award ***M0*** for attempt to solve quadratic equation.

(b) $(x-\alpha^2)(x-\beta^2)=x^2-(\alpha^2+\beta^2)x+\alpha^2\beta^2$ ***M1***

$x^2-5x+\left(-\frac{1}{2}\right)^2=0$ ***A1***

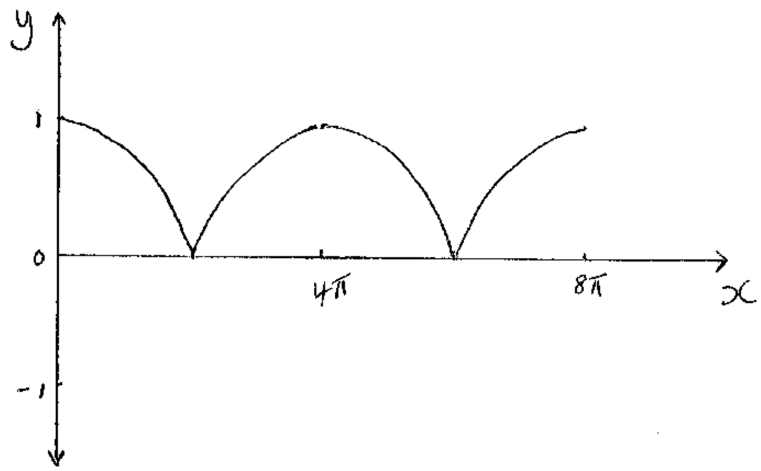
$x^2-5x+\frac{1}{4}=0$

Note: Final answer must be an equation. Accept alternative correct forms.

[2 marks]

Total [6 marks]

5. (a)



A1A1

Note: Award ***A1*** for correct shape and ***A1*** for correct domain and range.

[2 marks]

continued…

Question 5 continued

(b) $\left|\cos\left(\frac{x}{4}\right)\right| = \frac{1}{2}$

$x = \frac{4\pi}{3}$ ***A1***

attempting to find any other solutions ***M1***

Note: Award ***(M1)*** if at least one of the other solutions is correct (in radians or degrees) or clear use of symmetry is seen.

$x = 8\pi - \frac{4\pi}{3} = \frac{20\pi}{3}$

$x = 4\pi - \frac{4\pi}{3} = \frac{8\pi}{3}$

$x = 4\pi + \frac{4\pi}{3} = \frac{16\pi}{3}$ ***A1***

Note: Award ***A1*** for all other three solutions correct and no extra solutions.

Note: If working in degrees, then max ***A0M1A0***.

[3 marks]

Total [5 marks]

6. (a) $\overrightarrow{\mathrm{PR}} = \boldsymbol{a} + \boldsymbol{b}$ ***A1***

$\overrightarrow{\mathrm{QS}} = \boldsymbol{b} - \boldsymbol{a}$ ***A1***

[2 marks]

(b) $\overrightarrow{\mathrm{PR}} \cdot \overrightarrow{\mathrm{QS}} = (\boldsymbol{a} + \boldsymbol{b}) \cdot (\boldsymbol{b} - \boldsymbol{a})$ ***M1***

$= |\boldsymbol{b}|^2 - |\boldsymbol{a}|^2$ ***A1***

for a rhombus $|\boldsymbol{a}| = |\boldsymbol{b}|$ ***R1***

hence $|\boldsymbol{b}|^2 - |\boldsymbol{a}|^2 = 0$ ***A1***

Note: Do not award the final ***A1*** unless ***R1*** is awarded.

hence the diagonals intersect at right angles ***AG***

[4 marks]

Total [6 marks]

7. (a) **METHOD 1**

$$\frac{1}{2+3\text{i}}+\frac{1}{3+2\text{i}}=\frac{2-3\text{i}}{4+9}+\frac{3-2\text{i}}{9+4}$$ ***M1A1***

$$\frac{10}{w}=\frac{5-5\text{i}}{13}$$ ***A1***

$$w=\frac{130}{5-5\text{i}}$$

$$=\frac{130\times5\times(1+\text{i})}{50}$$

$$w=13+13\text{i}$$ ***A1***

[4 marks]

METHOD 2

$$\frac{1}{2+3\text{i}}+\frac{1}{3+2\text{i}}=\frac{3+2\text{i}+2+3\text{i}}{(2+3\text{i})(3+2\text{i})}$$ ***M1A1***

$$\frac{10}{w}=\frac{5+5\text{i}}{13\text{i}}$$ ***A1***

$$\frac{w}{10}=\frac{13\text{i}}{5+5\text{i}}$$

$$w=\frac{130\text{i}}{(5+5\text{i})}\times\frac{(5-5\text{i})}{(5-5\text{i})}$$

$$=\frac{650+650\text{i}}{50}$$

$$=13+13\text{i}$$ ***A1***

[4 marks]

(b) $w^*=13-13\text{i}$ ***A1***

$$z=\sqrt{338}e^{-\frac{\pi}{4}\text{i}}\left(=13\sqrt{2}\,e^{-\frac{\pi}{4}\text{i}}\right)$$ ***A1A1***

> **Note:** Accept $\theta=\frac{7\pi}{4}$.
> Do not accept answers for θ given in degrees.

[3 marks]

Total [7 marks]

8. (a) $1-2(2)=-3$ and $\frac{3}{4}(2-2)^2-3=-3$ ***A1***

both answers are the same, hence f is continuous (at $x=2$) ***R1***

Note: ***R1*** may be awarded for justification using a graph or referring to limits. Do not award ***A0R1***.

[2 marks]

(b) reflection in the y-axis

$$f(-x)=\begin{cases} 1+2x, & x\ge -2 \\ \frac{3}{4}(x+2)^2-3, & x<-2 \end{cases}$$ ***(M1)***

Note: Award ***M1*** for evidence of reflecting a graph in y-axis.

translation $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$

$$g(x)=\begin{cases} 2x-3, & x\ge 0 \\ \frac{3}{4}x^2-3, & x<0 \end{cases}$$ ***(M1)A1A1***

Note: Award ***(M1)*** for attempting to substitute $(x-2)$ for x, or translating a graph along positive x-axis.
Award ***A1*** for the correct domains (this mark can be awarded independent of the ***M1***).
Award ***A1*** for the correct expressions.

[4 marks]

Total [6 marks]

9. (a) $\sin x$, $\sin 2x$ and $4\sin x\cos^2 x$

$r = \frac{2\sin x\cos x}{\sin x} = 2\cos x$ ***A1***

Note: Accept $\frac{\sin 2x}{\sin x}$.

[1 mark]

(b) **EITHER**

$|r| < 1 \Rightarrow |2\cos x| < 1$ ***M1***

OR

$-1 < r < 1 \Rightarrow -1 < 2\cos x < 1$ ***M1***

THEN

$0 < \cos x < \frac{1}{2}$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$

$-\frac{\pi}{2} < x < -\frac{\pi}{3}$ or $\frac{\pi}{3} < x < \frac{\pi}{2}$ ***A1A1***

[3 marks]

(c) $S_\infty = \frac{\sin x}{1 - 2\cos x}$ ***M1***

$$S_\infty = \frac{\sin\left(\arccos\left(\frac{1}{4}\right)\right)}{1 - 2\cos\left(\arccos\left(\frac{1}{4}\right)\right)}$$

$= \frac{\frac{\sqrt{15}}{4}}{\frac{1}{2}}$ ***A1A1***

Note: Award ***A1*** for correct numerator and ***A1*** for correct denominator.

$= \frac{\sqrt{15}}{2}$ ***AG***

[3 marks]

Total [7marks]

10. $x = a\sec\theta$

$\frac{\mathrm{d}x}{\mathrm{d}\theta} = a\sec\theta\tan\theta$ ***(A1)***

new limits:

$x = a\sqrt{2} \Rightarrow \theta = \frac{\pi}{4}$ and $x = 2a \Rightarrow \theta = \frac{\pi}{3}$ ***(A1)***

$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{a\sec\theta\tan\theta}{a^3\sec^3\theta\sqrt{a^2\sec^2\theta - a^2}}\,\mathrm{d}\theta$ ***M1***

$= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\cos^2\theta}{a^3}\mathrm{d}\theta$ ***A1***

using $\cos^2\theta = \frac{1}{2}(\cos 2\theta + 1)$ ***M1***

$\frac{1}{2a^3}\left[\frac{1}{2}\sin 2\theta + \theta\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$ or equivalent ***A1***

$= \frac{1}{4a^3}\left(\frac{\sqrt{3}}{2} + \frac{2\pi}{3} - 1 - \frac{\pi}{2}\right)$ or equivalent ***A1***

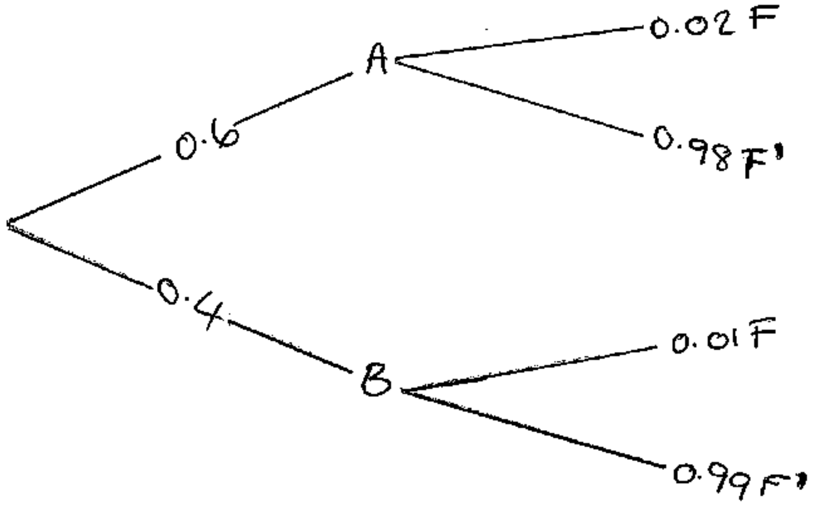
$= \frac{1}{24a^3}\left(3\sqrt{3} + \pi - 6\right)$ ***AG***

[7 marks]

Total [7 marks]

SECTION B

11. (a) (i)



A1A1

Note: Award *A1* for a correctly labelled tree diagram and *A1* for correct probabilities.

(ii) $\mathrm{P}(F) = 0.6 \times 0.02 + 0.4 \times 0.01$ ***(M1)***

$= 0.016$ ***A1***

(iii) $\mathrm{P}(A \mid F) = \dfrac{\mathrm{P}(A \cap F)}{\mathrm{P}(F)}$

$= \dfrac{0.6 \times 0.02}{0.016} \left(= \dfrac{0.012}{0.016} \right)$ ***M1***

$= 0.75$ ***A1***

[6 marks]

continued…

Question 11 continued

(b) (i) **METHOD 1**

$$\mathrm{P}(X=2)=\frac{{}^{3}C_{2}\times{}^{4}C_{1}}{{}^{7}C_{3}}$$ ***(M1)***

$$=\frac{12}{35}$$ ***A1***

METHOD 2

$$\frac{3}{7}\times\frac{2}{6}\times\frac{4}{5}\times 3$$ ***(M1)***

$$=\frac{12}{35}$$ ***A1***

(ii)

x	0	1	2	3
$\mathrm{P}(X=x)$	$\frac{4}{35}$	$\frac{18}{35}$	$\frac{12}{35}$	$\frac{1}{35}$

A2

Note: Award ***A1*** if $\frac{4}{35}$, $\frac{18}{35}$ or $\frac{1}{35}$ is obtained.

(iii) $\mathrm{E}(X)=\sum x\,\mathrm{P}(X=x)$

$$\mathrm{E}(X)=0\times\frac{4}{35}+1\times\frac{18}{35}+2\times\frac{12}{35}+3\times\frac{1}{35}$$ ***M1***

$$=\frac{45}{35}=\left(\frac{9}{7}\right)$$ ***A1***

[6 marks]

Total [12 marks]

12. (a) direction vector $\overrightarrow{AB} = \begin{pmatrix} 1 \\ 3 \\ -5 \end{pmatrix}$ or $\overrightarrow{BA} = \begin{pmatrix} -1 \\ -3 \\ 5 \end{pmatrix}$ ***A1***

$\boldsymbol{r} = \begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix} + t\begin{pmatrix} 1 \\ 3 \\ -5 \end{pmatrix}$ or $\boldsymbol{r} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + t\begin{pmatrix} 1 \\ 3 \\ -5 \end{pmatrix}$ or equivalent ***A1***

> **Note:** Do not award final ***A1*** unless '$r = \ldots$' (or equivalent) seen.
> Allow FT on direction vector for final ***A1***.

[2 marks]

(b) both lines expressed in parametric form:

L_1:
$x = 1 + t$
$y = 3t$
$z = 4 - 5t$

L_2:
$x = 1 + 3s$
$y = -2 + s$ ***M1A1***
$z = -2s + 1$

> **Notes:** Award ***M1*** for an attempt to convert L_2 from Cartesian to parametric form.
> Award ***A1*** for correct parametric equations for L_1 and L_2.
> Allow ***M1A1*** at this stage if same parameter is used in both lines.

attempt to solve simultaneously for x and y: ***M1***
$1 + t = 1 + 3s$
$3t = -2 + s$
$t = -\frac{3}{4}$, $s = -\frac{1}{4}$ ***A1***

substituting both values back into z values respectively gives $z = \frac{31}{4}$

and $z = \frac{3}{2}$ so a contradiction ***R1***

therefore L_1 and L_2 are skew lines ***AG***

[5 marks]

continued…

Question 12 continued

(c) finding the cross product:

$$\begin{pmatrix}1\\3\\-5\end{pmatrix}\times\begin{pmatrix}3\\1\\-2\end{pmatrix}$$ ***(M1)***

$= -\boldsymbol{i} - 13\boldsymbol{j} - 8\boldsymbol{k}$ ***A1***

> **Note:** Accept $\boldsymbol{i} + 13\boldsymbol{j} + 8\boldsymbol{k}$

$-1(0) - 13(1) - 8(-2) = 3$ ***(M1)***

$\Rightarrow -x - 13y - 8z = 3$ or equivalent ***A1***

[4 marks]

(d) (i) $$(\cos\theta =)\frac{\begin{pmatrix}k\\1\\-1\end{pmatrix}\cdot\begin{pmatrix}1\\1\\0\end{pmatrix}}{\sqrt{k^2+1+1}\times\sqrt{1+1}}$$ ***M1***

> **Note:** Award ***M1*** for an attempt to use angle between two vectors formula.

$$\frac{\sqrt{3}}{2} = \frac{k+1}{\sqrt{2(k^2+2)}}$$ ***A1***

obtaining the quadratic equation

$4(k+1)^2 = 6(k^2+2)$ ***M1***

$k^2 - 4k + 4 = 0$

$(k-2)^2 = 0$

$k = 2$ ***A1***

> **Note:** Award ***M1A0M1A0*** if $\cos 60^{\circ}$ is used $(k = 0$ or $k = -4)$.

continued…

Question 12 continued

(ii) $\boldsymbol{r} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$

substituting into the equation of the plane Π_2:

$3 + 2\lambda + \lambda = 12$ ***M1***

$\lambda = 3$ ***A1***

point P has the coordinates:

$(9, 3, -2)$ ***A1***

Notes: Accept $9\boldsymbol{i} + 3\boldsymbol{j} - 2\boldsymbol{k}$ and $\begin{pmatrix} 9 \\ 3 \\ -2 \end{pmatrix}$.

Do not allow FT if two values found for *k*.

[7 marks]

Total [18 marks]

13. (a) $f'(x) = \dfrac{(x^2+1) - 2x(x+1)}{(x^2+1)^2} \left(= \dfrac{-x^2 - 2x + 1}{(x^2+1)^2} \right)$ ***M1A1***

[2 marks]

(b) $\dfrac{-x^2 - 2x + 1}{(x^2+1)^2} = 0$

$x = -1 \pm \sqrt{2}$ ***A1***

[1 mark]

continued…

Question 13 continued

(c) $f''(x) = \dfrac{(-2x-2)(x^2+1)^2 - 2(2x)(x^2+1)(-x^2-2x+1)}{\left(x^2+1\right)^4}$ ***A1A1***

Note: Award ***A1*** for $(-2x-2)\left(x^2+1\right)^2$ or equivalent.

Note: Award ***A1*** for $-2(2x)\left(x^2+1\right)\left(-x^2-2x+1\right)$ or equivalent.

$= \dfrac{(-2x-2)\left(x^2+1\right) - 4x\left(-x^2-2x+1\right)}{\left(x^2+1\right)^3}$

$= \dfrac{2x^3+6x^2-6x-2}{\left(x^2+1\right)^3}$ ***A1***

$\left(= \dfrac{2\left(x^3+3x^2-3x-1\right)}{\left(x^2+1\right)^3}\right)$

[3 marks]

(d) recognition that $(x-1)$ is a factor ***(R1)***

$(x-1)\left(x^2+bx+c\right) = \left(x^3+3x^2-3x-1\right)$ ***M1***

$\Rightarrow x^2+4x+1=0$ ***A1***

$x = -2 \pm \sqrt{3}$ ***A1***

Note: Allow long division / synthetic division.

[4 marks]

(e) $\int_{-1}^{0} \dfrac{x+1}{x^2+1}\,\mathrm{d}x$ ***M1***

$\int \dfrac{x+1}{x^2+1}\,\mathrm{d}x = \int \dfrac{x}{x^2+1}\,\mathrm{d}x + \int \dfrac{1}{x^2+1}\,\mathrm{d}x$ ***M1***

$= \dfrac{1}{2}\ln\left(x^2+1\right) + \arctan(x)$ ***A1A1***

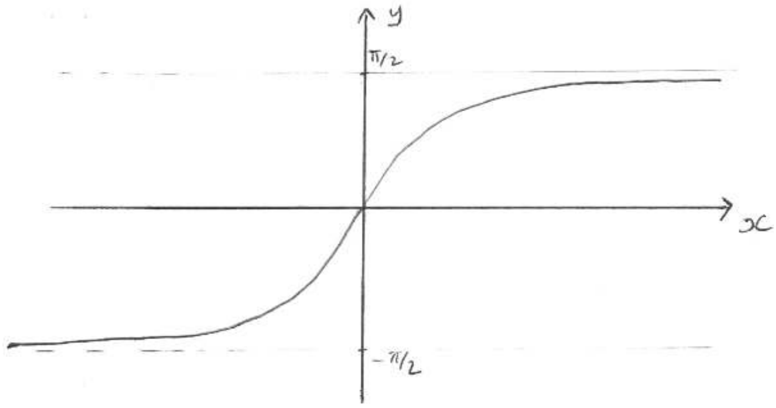
$= \left[\dfrac{1}{2}\ln\left(x^2+1\right) + \arctan(x)\right]_{-1}^{0} = \dfrac{1}{2}\ln 1 + \arctan 0 - \dfrac{1}{2}\ln 2 - \arctan(-1)$ ***M1***

$= \dfrac{\pi}{4} - \ln\sqrt{2}$ ***A1***

[6 marks]

Total [16 marks]

14. (a)



A1A1

> **Note:** ***A1*** for correct shape, ***A1*** for asymptotic behaviour at $y = \pm\frac{\pi}{2}$.

[2 marks]

(b) $h \circ g(x) = \arctan\left(\frac{1}{x}\right)$ ***A1***

domain of $h \circ g$ is equal to the domain of g : $x \in \mathbb{R}$, $x \neq 0$ ***A1***

[2 marks]

(c) (i) $f(x) = \arctan(x) + \arctan\left(\frac{1}{x}\right)$

$$f'(x) = \frac{1}{1+x^2} + \frac{1}{1+\frac{1}{x^2}} \times -\frac{1}{x^2}$$ ***M1A1***

$$f'(x) = \frac{1}{1+x^2} + \frac{-\frac{1}{x^2}}{\frac{x^2+1}{x^2}}$$ ***(A1)***

$$= \frac{1}{1+x^2} - \frac{1}{1+x^2}$$

$$= 0$$ ***A1***

continued…

Question 14 continued

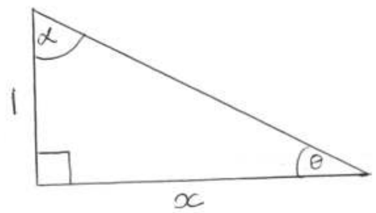
(ii) **METHOD 1**

f is a constant **R1**
when $x > 0$

$f(1) = \frac{\pi}{4} + \frac{\pi}{4}$ **M1A1**

$= \frac{\pi}{2}$ **AG**

METHOD 2



from diagram

$\theta = \arctan\frac{1}{x}$ **A1**

$\alpha = \arctan x$ **A1**

$\theta + \alpha = \frac{\pi}{2}$ **R1**

hence $f(x) = \frac{\pi}{2}$ **AG**

METHOD 3

$\tan(f(x)) = \tan\left(\arctan(x) + \arctan\left(\frac{1}{x}\right)\right)$ **M1**

$= \frac{x + \frac{1}{x}}{1 - x\left(\frac{1}{x}\right)}$ **A1**

denominator $= 0$, so $f(x) = \frac{\pi}{2}$ (for $x > 0$) **R1**

[7 marks]

continued…

Question 14 continued

(d) (i) Nigel is correct. ***A1***

METHOD 1

$\arctan(x)$ is an odd function and $\frac{1}{x}$ is an odd function

composition of two odd functions is an odd function and sum of two odd functions is an odd function ***R1***

METHOD 2

$$f(-x) = \arctan(-x) + \arctan\left(-\frac{1}{x}\right) = -\arctan(x) - \arctan\left(\frac{1}{x}\right) = -f(x)$$

therefore f is an odd function. ***R1***

(ii) $f(x) = -\frac{\pi}{2}$ ***A1***

[3 marks]

Total [14 marks]